ENGN 810 Homework 5

NAME AS IT APPEARS IN BANNER

Due: 11/26/18

Problem 1: 6.41

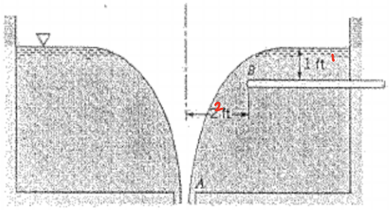

FIGURE P6.55

Given: no air in at B

Find: strength, Γ

$$V_\theta = \frac{\Gamma}{2\pi r}$$

$$\cancel{P_1} + \tfrac{1}{2}V_1^2\rho + \rho g h_1 = \cancel{P_2} + \tfrac{1}{2}V_2^2\rho + \rho g h_2$$

$=0 \qquad =0$

$$\tfrac{1}{2}V_1^2\rho + \cancel{\rho} g(1\,ft) = \tfrac{1}{2}V_2^2\cancel{\rho}$$

$$\cancel{\tfrac{1}{2}V_1^2} + g(ft) = \tfrac{1}{2}V_2^2$$

$V_1 = 0$

$$V_2 = \frac{\Gamma}{2\pi r} = V_\theta$$

$$g\cdot(1\,ft) = \frac{1}{2}\frac{\Gamma^2}{4\pi^2 r^2}$$

$$\Gamma^2 = g(1\,ft)\cdot 8\pi^2 r^2$$

$$\Gamma = \left(33.2\tfrac{ft}{s^2}\right)(1\,ft)\,\pi^2\,8\,(2\,ft)^2$$

$$\boxed{\Gamma = 102\ \tfrac{ft^2}{s}}$$

Problem 2 6.42

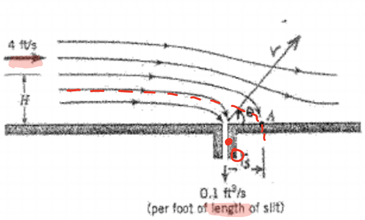

FIGURE P6.56

Given:

Uniform flow + Sink

Find: Stagnation point,

H such that fluid does not go in sink

$$\psi_{uniform} = Uy \qquad \psi_{sink} = \frac{-m\theta}{2\pi}$$

$$\psi = Uy - \frac{m\theta}{2\pi}$$

$$y = r\sin\theta$$

$$\psi = Ur\sin\theta - \frac{m\theta}{2\pi}$$

Along Wall A:

$r = 0 \to \infty, \quad \theta = 0$

@ Stagnation point $\vec{v} = 0$

$$V_r = U\cos\theta - \frac{m}{2\pi r} = 0$$

$$U\cos(0) = \frac{m}{2\pi r}$$

$$4\tfrac{ft}{s}(1) = \frac{0.2\ ft^3/s}{2\pi\ r} \qquad m = 0.2\tfrac{ft^2}{s}$$

$$r = \frac{0.2\ ft^3/s}{2\pi\,(4\tfrac{ft}{s})} = 0.00796\ ft$$

Velocity components:

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta - \frac{m}{2\pi r} \qquad V_\theta = -\frac{\partial\psi}{\partial r} = -U\sin\theta$$

@ $\theta = 0$, $r = 0.00796$, $\psi = Ur\sin\theta - \frac{m\theta}{2\pi} = 0$

↳ stagnation streamline

$$Ur\sin\theta = \frac{m\theta}{2\pi}$$

$$r\sin\theta = \frac{m\theta}{2\pi U} = y$$

$H = y$ as $\theta \to \pi$

$$H = \frac{m\pi}{2\pi U} = \frac{m}{2U} = \frac{0.2\ ft^2/s}{2\cdot 4\ ft/s} = 0.025\ ft$$

$$\boxed{\text{Stagnation point @ } r = 0.00796\,ft,\ \theta = 0}$$

$$\boxed{H = 0.025\ ft}$$

Problem 3 6.43

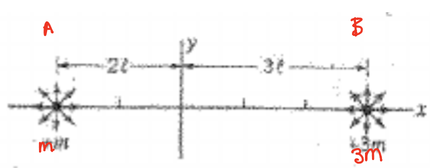

FIGURE P6.57

Find: Stagnation pt, x

$$\psi_{source} = \frac{m\theta}{2\pi} \qquad \psi = \overbrace{\frac{m\theta}{2\pi}}^{A} + \overbrace{\frac{3m\theta}{2\pi}}^{B}$$

$$V_{r_A} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{m}{2\pi r_A} \qquad V_{r_B} = \frac{3m}{2\pi r_B}$$

@ Stagnation pt, $V_{rA} = V_{rB}$

$$\frac{2\pi}{m}\cdot\frac{m}{2\pi r_A} = \frac{3m}{2\pi r_B}\cdot\frac{2\pi}{m}$$

$$3r_A = r_B$$

$\Leftarrow$ $r_A = (2\ell + x) \qquad r_B = (3\ell - x)$

$$3(2\ell + x) = 3\ell - x$$

$$6\ell + 3x = 3\ell - x$$

$$3\ell = -4x$$

$$x = -\tfrac{3}{4}\ell$$

$\Leftarrow$ $\boxed{x = -\tfrac{3}{4}\ell}$

Problem 4: 6.44

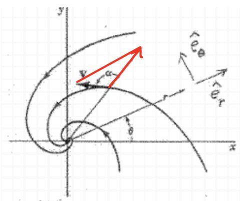

FIGURE P6.58

$$\phi = \frac{\Gamma}{2\pi}\theta - \frac{m}{2\pi}\ln r$$

Show angle α b/t $\vec{V}$ and $\hat{r}$ is const.

$$V_r = \frac{\partial \phi}{\partial r} = \frac{m}{2\pi r} \qquad V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{\Gamma}{2\pi r}$$

$$\vec{V} = \frac{m}{2\pi r}\hat{r} + \frac{\Gamma}{2\pi r}\hat{\theta}$$

angle, α:

$$\cos\alpha = \frac{\vec{V}\cdot\hat{r}}{|V|} = \frac{\frac{m}{2\pi r}}{\sqrt{\frac{m^2}{4\pi^2 r^2} + \frac{\Gamma^2}{4\pi^2 r^2}}} = \frac{\frac{m}{2\pi r}}{\sqrt{\frac{m^2+\Gamma^2}{4\pi^2 r^2}}} = \frac{m}{2\pi r}\cdot\frac{2\pi r}{\sqrt{m^2+\Gamma^2}}$$

$$\cos\alpha = \frac{m}{(m^2+\Gamma^2)^{\frac{1}{2}}} \Rightarrow \alpha = \cos^{-1}\left[\frac{m}{\sqrt{m^2+\Gamma^2}}\right]$$

b/c m, Γ are const, $\underline{\alpha \text{ must be const.}}$

Problem 5: 6.51

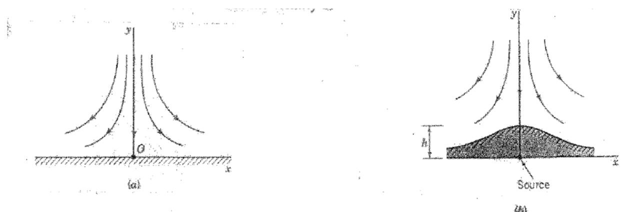

FIGURE P6.62

$\psi = Axy$ Find: relationship b/t h, A, and m

$$\psi = Axy + \frac{m\theta}{2\pi}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\psi = A r^2\cos\theta\sin\theta + \frac{m\theta}{2\pi}$$

$$= \frac{A}{2}r^2\cos\theta\, 2\sin\theta + \frac{m\theta}{2\pi} = \frac{1}{2}Ar^2\sin 2\theta + \frac{m\theta}{2\pi}$$

$$V_\theta = \frac{-\partial\psi}{\partial r} = -A2r\cos\theta\sin\theta = -A\sin 2\theta$$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\frac{1}{2}Ar^2\, 2\cos 2\theta + \frac{m}{2\pi r} = Ar\cos 2\theta + \frac{m}{2\pi r}$$

@ Stagnation point, $V=0$, $\theta = \frac{\pi}{2}$, $r=h$

$$V_\theta = 0 = -A\sin(\pi) = 0$$

$$V_r = A(h)\cos(\pi) + \frac{m}{2\pi h} = 0$$

$$-Ah = \frac{-m}{2\pi h}$$

$$\boxed{2\pi A h^2 = m}$$

Problem 6: 6.64

Find: F_{TOT}

eq. 6.124: $F_y = -\rho U\Gamma$ $D = 9\text{ ft}$ $L = 50\text{ ft}$

$$\vec{\omega} = \frac{750\text{ rev}}{\text{min}} = \frac{750\,(2\pi)\text{ rad}}{60\text{s}} = 78.5\ \frac{\text{rad}}{\text{s}}$$

a) $U = 10\text{ mph}$

$\vec{V} = r\omega\hat{e}_\theta$

$d\vec{s} = r\,d\theta\,\hat{e}_\theta$

$V = \omega r$

$$\Gamma = \oint \vec{V}\cdot d\vec{s}$$

$$= \int_0^{2\pi} r\omega\,\hat{e}_\theta \cdot r\,d\theta\,\hat{e}_\theta$$

$$= \int_0^{2\pi} r^2\omega\,d\theta$$

$$= r^2\omega\,2\pi = (4.5\text{ft})^2\left(78.5\,\frac{\text{rad}}{\text{s}}\right)2\pi = 9988\ \frac{\text{ft}^2}{\text{s}}$$

$$U = 10\text{ mph}\cdot\frac{5280\text{ ft}}{1\text{ mile}}\cdot\frac{1\text{ hr}}{3600\text{ s}} = 14.67\ \frac{\text{ft}}{\text{s}}$$

$$F_y = 2\text{ cylinders}\cdot(50)\,(0.00238)\left(9988\,\frac{\text{ft}^2}{\text{s}}\right)\left(14.67\,\frac{\text{ft}}{\text{s}}\right)$$

$$\boxed{F_y = 34{,}872\text{ lbs}}$$

b) $\vec{V} = 3\cdot V$ from part A $\therefore$ $F_y = 3\cdot F_y$ from A

$$\boxed{F_y = 104{,}620\text{ lbs}}$$